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# DROPWISE CONDENSATION THEORY

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**Abstract**—Improved theoretical results for heat transfer through individual drops and for the mean distribution of drop sizes are used as a basis for assessing the validity of the basic assumption of dropwise condensation theory [1] i.e. that the mean heat flux can be found from steady calculation of the heat transfer through individual drops and a steady distribution of drop sizes.

## NOMENCLATURE

- $A(r)dr$, fraction of surface covered by drops with radii in the interval $r$, $r + dr$;
- $f$, parameter in drop size distribution theory [7];
- $h_{fg}$, specific enthalpy of liquid–vapour phase change;
- $k_f$, thermal conductivity of drop;
- $k_p$, thermal conductivity of promoter layer;
- $m$, entier $\{\ln(r/\hat{r})/\ln\gamma\}$;
- $N$, $Q_c r/k_f\Delta T$ (excluding effect of surface curvature);
  $Q_c r/k_f\,(\Delta T - 2\sigma v_f T_v/h_{fg} r)$ (including effect of surface curvature);
- $n$, ratio of principal specific heat capacities $c_P/c_v$;
- $Q$, mean heat flux for condensing surface;
- $Q_b$, mean heat flux through base of a drop;
- $Q_c$, mean heat flux through curved surface of a drop;
- $R$, specific ideal-gas constant;
- $R_p$, thermal resistance of promoter;
- $r$, drop radius;
- $r_s$, effective minimum drop radius;
- $\hat{r}$, effective maximum drop radius;
- $\check{r}$, radius of smallest viable drop, $2\sigma v_f T_v/h_{fg}\Delta T$;
- $s$, distance between neighboring nucleation sites;
- $T_v$, vapour temperature;
- $t_p$, promoter layer thickness;
- $v_f$, specific volume of condensate;
- $v_g$, specific volume of vapour;
- $\alpha_i$, interface heat-transfer coefficient;
- $\gamma$, parameter in drop size distribution theory [7];
- $\Delta T$, vapour-to-surface temperature difference;
- $v$, $\alpha_i r/k_f$;
- $\sigma$, surface tension.

## INTRODUCTION

THE BASIC assumption of the theory of dropwise condensation [1] is that the mean heat flux for the condensing surface may be obtained from a calculation of the *steady* heat-transfer rate for a drop of given size and a *steady* distribution of drop sizes. In view of the highly non-steady nature of the actual process, wherein around a million coalescences can occur in one second on a square centimetre of the condensing surface, this procedure may seem somewhat dubious.

The steady model has been justified [2] by the excellent agreement between the theoretical result and heat-transfer measurements. Such comparisons are, however, somewhat confused by additional assumptions and approximations incorporated in the theory [1]. In the original theory an approximate equation was used for the heat-transfer rate through a drop of given size and this was used in conjunction with an approximate drop size distribution to evaluate the mean heat flux for the surface. The object of the present work was to examine more carefully the validity of the basic assumption by adopting more rigorous theoretical solutions for heat transfer through a single drop and for the drop size distribution.

## HEAT TRANSFER THROUGH A SINGLE DROP

For a hemispherical drop with uniform base temperature, the combined resistance arising from conduction in the drop and interphase mass transfer at the vapour–liquid interface has been evaluated [3]. More recently solutions have been obtained for other contact angles [4, 5]. For the case of the hemispherical drop the result may be expressed

$$N = v - v^2 \sum_{m=0}^{\infty} \left\{ \left( \frac{4m+3}{2m+1+v} \right) \times \left( \frac{1}{2m+1} \right)^2 \left( \frac{1}{2} \times \frac{3}{4} \times \cdots \frac{2m+1}{2m+2} \right)^2 \right\} \quad (1)$$

where

$$N = \frac{Q_c r}{k_f \Delta T} \quad (2)$$

$$v = \frac{\alpha_i r}{k_f} \quad (3)$$

and $\alpha_i$ is the interface heat-transfer coefficient which,

for a given vapour pressure, may be estimated from kinetic theory. Equation (1) may be approximated,* for all $v$, by

$$N = \frac{2}{\pi}\ln\left\{1 + v\left(1.09 + \frac{\pi/2 - 1.09}{v/5.7 + 1}\right)\right\}. \quad (4)$$

Equation (4) has a maximum error less than 0.3%.

The effect of surface curvature is also included by replacing $\Delta T$ in equation (2) by $\Delta T - 2\sigma v_f T_v/h_{fg}r$. Then, from equations (2) and (4) we obtain the mean heat flux through the base of a hemispherical drop:

$$Q_b = 2Q_c = \frac{4}{\pi}\frac{k_f}{r}\left(\Delta T - \frac{2\sigma v_f T_v}{h_{fg}r}\right) \times \ln\left\{1 + v\left(1.09 + \frac{\pi/2 - 1.09}{v/5.7 - 1}\right)\right\}. \quad (5)$$

## DROP SIZE DISTRIBUTION

A theoretical calculation [7] of the mean distribution of drop sizes, developed since the original theory [1], gives

$$A(r) = \frac{f}{\hat{r}}\left[1 + 2\sum_{i=1}^{m}\left\{\left(\frac{1-f}{\gamma}\right)^i\left(1 - \frac{r}{\hat{r}\gamma^i}\right)\right\}\right] \quad (6)$$

where $A(r)dr$ is the fractional area covered by drops having base radius in the range, $r, r + dr$. The values of the constants $f$ and $\gamma$ were calculated† [7] as 0.55 ± 0.05 and 0.19 ± 0.03 respectively, and $m$ is the largest integer for which $\gamma^m \geq r/\hat{r}$, i.e. $m = \text{entier}\{\ln(r/\hat{r})/\ln\gamma\}$.

It has been shown [6] that equation (6) may be satisfactorily approximated by

$$A(r) = \frac{1}{r}\left\{0.871\left(\frac{r}{\hat{r}}\right)^{1/2} - 1.39\left(\frac{r}{\hat{r}}\right) + 1.296\left(\frac{r}{\hat{r}}\right)^2\right\}. \quad (7)$$

## HEAT FLUX–TEMPERATURE DIFFERENCE RELATIONSHIP

The mean heat flux is given by

$$Q = \int_{r_v}^{\hat{r}} Q_b A(r)dr. \quad (8)$$

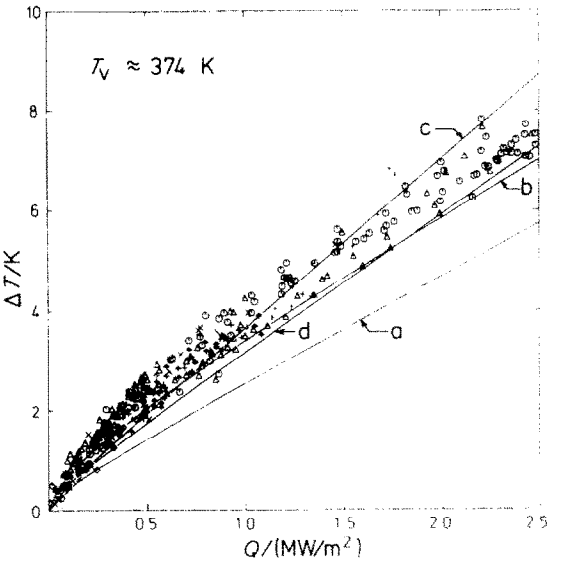
Thus, for given values of smallest and largest drop radius and adopting equations (5) and (7), we may obtain, wholly theoretically, the dependence of $Q$ on $\Delta T$.

## COMPARISON WITH EXPERIMENT

The radius of the smallest viable drop is $\check{r} = 2\sigma v_f T_v/h_{fg}\Delta T$ while, for condensation of steam, visual observation suggests an average maximum adherent drop radius of around 1 mm. Using these values for the lower and upper limits the integral has been evaluated numerically for a range of values of $\Delta T$ and for vapour temperatures at which heat-transfer measurements have been made. As in [1] we have used

$$\alpha_i = \left(\frac{n-1}{n+1}\right)\frac{h_{fg}^2}{v_g T_v}\left(\frac{2\pi}{RT_v}\right)^{1/2}. \quad (9)$$

In Fig. 1 the result (line a) is compared with experimental data for steam at pressures near atmospheric and around 5 kPa (0.05 bar). In view of the fact that the theoretical result is wholly free of empiricism and contains no adjustable parameters the comparison is considered generally satisfactory. It is seen, however, that, for both pressures, the theory gives an



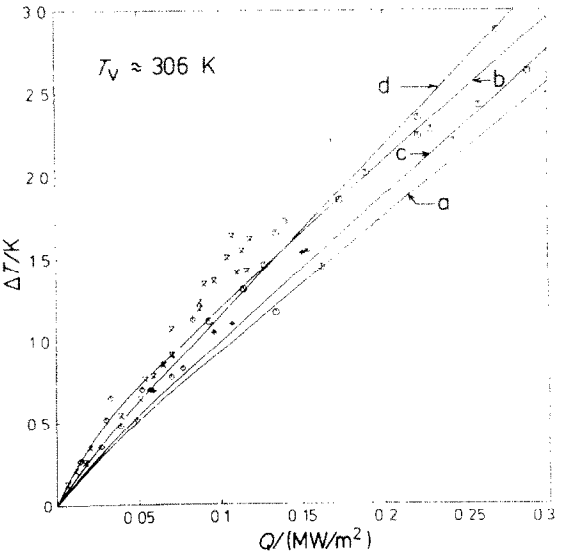




FIG. 1. Dropwise condensation of steam at atmospheric and low pressure. Comparison between experiment and theory. ◇, [8, 10]; +, [9]; △, [11]; ⏃, [12]; ⧖, [13]; ×, [14]; ○, [15].

* In obtaining equation (4) use has been made of the limiting results [6]:
$N \simeq v - 0.8v^2$, for small $v$;
$N \simeq (2/\pi)\ln(1 + 1.09v)$, for large $v$.

† New calculations, made in the course of the present work, suggest $f = 0.59$, $\gamma = 0.164$.

overestimate of the heat flux. This suggests that, notwithstanding the high rates of coalescence, the steady conduction treatment of heat transfer through the drops is probably satisfactory. One would expect that disturbances following coalescence would result in enhancement of the heat transfer so that a steady conduction calculation would *underestimate* the heat-transfer rate.

A possible explanation of the discrepancy between the theoretical calculation and the observations is the fact that the theoretical drop size distribution [7] relates only to those drops which have undergone coalescence. The theoretical distribution would seriously overestimate the population of the primary drops (those which form at nucleation sites and have not yet undergone coalescence) if the mean spacing of nucleation sites significantly exceeds the smallest drop diameter. Estimates of the nucleation site density [13, 16–18] based on optical and electron microscope photographs, however, suggest that the primary drops are closely packed. Graham and Griffith [13, 16] give nucleation site densities of $2 \times 10^8\ \mathrm{cm}^{-2}$ and $6 \times 10^8\ \mathrm{cm}^{-2}$ for conditions for which $\check{r} \simeq 0.07\ \mu\mathrm{m}$, while Tanasawa and co-workers [17, 18] indicate a site density exceeding $10^{10}\ \mathrm{cm}^{-2}$ when $\check{r} \simeq 0.01\ \mu\mathrm{m}$. These data suggest a mean spacing-to-radius ratio $s/\check{r}$ for nucleation sites, of around 7 (on the basis of a uniform equilateral triangular array of sites one would estimate a site density of $2/s^2\sqrt{3}$, so that when $s/r = 7$ we obtain site densities of $4.8 \times 10^8\ \mathrm{cm}^{-2}$ and $2.4 \times 10^{10}\ \mathrm{cm}^{-2}$ using values of $0.07\ \mu\mathrm{m}$ and $0.01\ \mu\mathrm{m}$ respectively for $\check{r}$), so that primary droplets grow on average to a radius of about $3\check{r}$ to $4\check{r}$ before coalescing with neighbours. A lower limit to the validity of the theoretical drop size distribution might thus be around $5\check{r}$.

The sensitivity of the heat flux calculation to the (erroneous) lower end of the drop size distribution can be assessed by evaluating the integral in equation (8) using a lower limit in excess of $\check{r}$. This procedure excludes from the calculation heat transfer through drops with radius between $\check{r}$ and the lower limit of integration on the basis that there are, in fact, many fewer of these drops than indicated by equation (7). Results were obtained using various multiples of $\check{r}$ as the lower limit of integration. It was found that agreement with the experimental data was best when using a lower limit of around $10\check{r}$. The result obtained when using a value of $10\check{r}$ is shown (line b) in Fig. 1.

The fact that different promoters are known to give significantly different results (see for instance [9]) suggests that a promoter layer resistance (not included in the present calculation) might play a role in explaining the discrepancy between the theoretical result and the experimental data. In order to investigate this possibility $\Delta T$ in equation (5) was replaced by $\Delta T - Q_b R_p$ where $R_p$ represents the promoter layer resistance (if $Q_b$ were uniform over the base of the drop and if the additional resistance were due simply to conduction in the promoter layer then $R_p = t_p/k_p$) and the equation re-arranged to give an amended expression for $Q_b$. Various values of $R_p$ were used and the integral in equation (8) evaluated (using $\check{r}$ as the lower limit). It was found that fair agreement with the experimental data could be obtained when using $R_p = 0.15\ \mathrm{m}^2\ \mathrm{K/MW}$, see line c in Fig. 1. If it is assumed that $R_p$ represents the resistance due to conduction in a promoter layer having, for example, a thermal conductivity equal to that of paraffin wax (0.25 W/m K), this would represent a layer thickness of about 0.04 $\mu$m, i.e. around 20 times the thickness of a mono-molecular layer of the promoter dioctadecyl disulphide.

Finally, it should be noted that, in all of the foregoing calculations, the maximum drop radius was taken as 1 mm. Further calculations were carried out (taking the lower limit of integration in equation (8) as $\check{r}$ and not including allowance for promoter layer resistance) using different values of $\hat{r}$. It was found that a value of $\hat{r}$ of 1.5 mm led to good agreement with the experimental data at low pressure but continued to overestimate the heat flux at atmospheric pressure, while a value of 2.0 mm gave good agreement at atmospheric pressure but under-estimated the heat flux at low pressure. Shown on Fig. 1, line d, are the results obtained with $\hat{r} = 1.75$ mm.

## CONCLUDING REMARKS

1. A wholly theoretical calculation, based on steady conduction in each drop and including the resistance associated with interphase mass transfer and the effect of surface curvature, together with a steady distribution of drop sizes, gives fair agreement with experimental data at atmospheric and low pressure.

2. The fact that the theoretical result *overestimates* the heat transfer suggests that disturbances in the drops following coalescence do not play a significant role, i.e. despite the fact that the time interval, during which drops grow without encountering drops of similar size, is very short, the time taken for disturbances to die away, following coalescence, is significantly shorter.

3. The magnitude of the discrepancy between the theoretical result and experiment is put in perspective by the fact that it may be approximately accounted for by:

(a) an overestimate of the population of the smallest (primary) drops approximately as anticipated, or

(b) an additional resistance equal to that of a promoter having a thickness of about 20 molecular layers, or

(c) by taking, as the maximum drop radius, a value of about 1.75 mm rather than 1 mm.

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## THEORIE DE LA CONDENSATION EN GOUTTES

**Resume**—Des resultats théoriques connus sur le transfert de chaleur entre les gouttes individuelles et sur la distribution moyenne des trailles de gouttes sont utilisés pour évaluer la validité de l'hypothèse à la base de la théorie de la condensation en gouttes [1], c'est-à-dire que le flux de chaleur moyen peut être trouvé à partir du calcul en régime stationnaire du transfert thermique à travers les gouttes individuelles et d'une distribution stationnaire des tailles des gouttes.

## EINE THEORIE FÜR TROPFENKONDENSATION

**Zusammenfassung** — Verbesserte theoretische Beziehungen für den Wärmedurchgang durch einzelne Tropfen und für die mittlere Tropfengrößenverteilung werden dazu benutzt, die Gültigkeit der grundlegenden Annahme der Tropfenkondensationstheorie [1] abzuschätzen, d.h. daß der mittlere Wärmestrom aus der stationären Berechnung des Wärmedurchgangs durch einzelne Tropfen bei zeitlich unveränderlicher Tropfengrößenverteilung ermittelt werden kann.

## ТЕОРИЯ КАПЕЛЬНОЙ КОНДЕНСАЦИИ

**Аннотация** — На основании более точных теоретических результатов по переносу тепла через отдельные капли и среднему распределению капель по размерам проведена оценка справедливости основного допущения теории капельной конденсации, согласно которому среднюю плотность теплового потока можно определить из расчета переноса тепла через отдельные капли и распределения капель по размерам в стационарном режиме.